AD A075472

LEVEL II

(12)

(14) TR-79-6

TECHNICAL REPORT

(11) SEPTEMBER 1979

(9) Research rept. 1 Sep 78 - 1 Sep 79,

(12) 51

(6)

THEORY AND PRACTICE FOR THE USE OF CUT SCORES FOR PERSONNEL DECISIONS.

BY

(10) DAVID T. CHUANG, JAMES J. CHEN AND MELVIN R. NOVICK

THE UNIVERSITY OF IOWA

REPORT PREPARED UNDER OFFICE OF NAVAL RESEARCH

CONTRACT (15) N00014-77-C-0428

MELVIN R. NOVICK, PRINCIPAL INVESTIGATOR

UNIVERSITY OF IOWA

IOWA CITY, IOWA

(16) RR04204 (17) RR0420401

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report 79-6	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) "THEORY AND PRACTICE FOR THE USE OF CUT-SCORES FOR PERSONNEL DECISIONS"		5. TYPE OF REPORT & PERIOD COVERED Research Report Sept.1,1978 to Sept.1,1979
		6. PERFORMING ORG. REPORT NUMBER Technical Report 79-6
7. AUTHOR(s) David T. Chuang, James J. Chen, and Melvin R. Novick		8. CONTRACT OR GRANT NUMBER(s) #00014-77-C-0428
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Iowa Division of Educational Psychology Iowa City, Iowa 52242		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 6115 3N; RR 042-04; RR042-04-01; NR 150-404
11. CONTROLLING OFFICE NAME AND ADDRESS Personnel and Training Research Programs Office of Naval Research (Code 458) Arlington, Virginia 22217		12. REPORT DATE September, 1979
		13. NUMBER OF PAGES 38
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Cut-scores, personnel decisions, posterior ratios, monitone likelihood ratios

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper provides necessary and sufficient conditions for the existence of a cut-score for arbitrary utility functions. Classical theories of monitone likelihood ratios and stochastically increasing distribution are extended to provide the necessary theory of monitone posterior ratios, which provides a sufficiency condition. Necessity is studied in individual cases. In general, only modest sample sizes and avoidance of situations with extreme cut-scores is necessary.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

411417

Theory and Practice For the Use of Cut Scores for Personnel Decisions

David T. Chuang, James J. Chen and Melvin R. Novick

The University of Iowa

Cut-scores are commonly used in industrial personnel selection, academic selection, minimum competence certification testing, and professional licensing, using simple and multiple-person/multiple-job category decision paradigms. Previous approaches have proposed cut-score solutions in a variety of applications (Gross and Su, 1975; Novick and Lindley, 1978, 1979; Petersen, 1976; and Van der Linden and Mellenbergh, 1977) using threshold , normal ogive, linear and discrete utility functions. This paper considers these results by investigating conditions on the posterior, likelihood and utility functions required for a cut-score to be valid. The result is the showing that cut-scores are appropriate in a wide range of applications, but they are less than universally appropriate. Following this a general paradigm and computational algorithm for cut-score solutions is developed under the assumption that the conditions for a cut-score have been satisfied.

Introduction

In a decision problem, a Bayesian has a probability distribution over the states of nature and a utility function over actions and states. A Bayesian makes a decision by maximizing expected utility. Applications of Bayesian decision theory to personnel selection have recently been studied by Gross and Su (1975), Novick and Petersen (1976), Novick and Lindley (1978, 1979), Petersen (1976), Petersen and Novick (1976) and Van der Linden and Mellenbergh (1977). All except Novick and Lindley (1978, 1979) assume utility functions are threshold or linear, and the likelihood functions used are also limited. In this paper we study personnel decision problems using more general utility and likelihood functions.

In section one of this paper we discuss the likelihood functions and prior distributions we shall use. We show that the stochastic increasing (SI) property for the posterior distribution is a necessary and sufficient condition for a cut-score for an arbitrary utility function. As a special case, we study likelihood functions satisfying monotone likelihood ratio (MLR), a condition studied extensively in Karlin and Rubin (1956), Karlin (1957a, 1957b), Lehmann (1959) and DeGroot (1970). We define the monotone posterior ratio (MPR) property and show that it implies the stochastically increasing property; thus it implies a cut-score for an arbitrary utility function. We also show that MLR implies MPR with any prior distribution.

In section two we note that most posterior and predictive densities do not satisfy the MPR or SI properties, but show that in important cases cut-scores are appropriate provided that discrimination is not required in the tails of the distribution. In particular we carefully study the posterior "t" distribution obtained with many Bayesian normal models.

In section three we provide a very general multi-person/multi-job category paradigm appropriate for a wide variety selection, guidance and classification applications. By assuming the appropriateness of cut-scores based on the theory and restrictions of section 2 we are able to provide simple computational algorithms for the general paradigm.

In this paper, we use the following notation. We use N to denote the total number of applicants. θ is the parameter we are interested in and $T(x_i)$ is the test score of the ith applicant. A job performance score is denoted by y_i. In one context we use A,B,C,....R to represent the names of jobs that have openings. $U_A(\theta)$, $U_B(\theta)$, $U_C(\theta)$, ...$U_R(\theta)$ are the utilities of assigning an applicant with ability θ to jobs A,B,C,... and R respectively. The expected utility of assigning a person with test score T(x) to a job, say A, is $E(U_A(\theta))$, which is a function of T(x). The indicated expectation is with respect to the Bayes distribution for θ. We let $V_A(T(x)) \equiv E(U_A(\theta))$. This notation emphasizes that V does not depend on θ. An optimal decision procedure is a procedure that maximizes the sum of expected utilities of the selection across all persons to be selected. In another context the utility function is defined over the performance scores y. The utilities are then $U_A(y)$,..., $U_R(y)$ and expectations are taken with respect to the posterior predictive distribution for y. All results are valid whether the focus is on the unobservable ability (θ) or the observable performance y.

Likelihood Function and Prior Distribution

Let θ be a person's ability (or a measure of performance), and $T(x)$ be his test score. It may be reasonable to desire that if one person has a higher test score than another then the statistical analysis will provide some evidence that the first person's ability is not less than the second, all else being equal. Thus it may be reasonable to desire that the densities $p(x|\theta)$ have monotone likelihood ratio (MLR).

Definition 1: The real parameter family of densities $p(x|\theta)$ is said to have monotone likelihood ratio if there exists a real-valued function $T(x)$ such that for any $\theta_1 < \theta_2$ the likelihood functions $p(x|\theta_2)$ and $p(x|\theta_1)$ are distinct and the ratio $p(x|\theta_2)/p(x|\theta_1)$ is a non-decreasing function of $T(x)$. The MLR condition is a particular technical requirement which implies that increasing values of $T(x)$ will provide increasing evidence from the likelihood that θ_2 is greater than θ_1 when in fact $\theta_2 > \theta_1$.

It can be shown that the exponential family, (normal, binomial, Poisson, exponential, etc.) and some other widely used distributions have MLR. This condition is not satisfied by all distributions (e.g. the predictive distribution of Student's t from linear regression); however, in such cases it may be except for extreme values of $T(x)$. Monotone likelihood ratio is generally an easy condition to verify.

From a decision theory point of view, we consider θ as a random variable and define a similar ratio monotonicity condition for such distributions.

Definition 2: The class of posterior densities $p(\theta|x)$ is said to have monotone posterior ratio (MPR) if there exists a real valued function $T(x)$ such that for any $T(x_1) < T(x_2)$ the posterior distributions $p(\theta|T(x_1))$ and $p(\theta|T(x_2))$ are distinct and the ratio $p(\theta|T(x_2))/p(\theta|T(x_1))$ is a non-decreasing function of θ.

We can see definition 2 is exactly the same as definition 1 except the roles of θ and $T(x)$ are exchanged. *The suggestion here is that for* any prior distribution, higher values of $T(x)$ will be suggestive of higher values for θ in the posterior distribution. The following lemma establishes this condition for MPR.

Lemma 1: Suppose $p(\theta)$ is any prior distribution and $f(x|\theta)$ has MLR in the statistic $T(x)$, then the posterior distribution $p(\theta|x)$ of θ has MPR with statistic $T(x)$.

Proof: By definition $p(\theta|x) = \dfrac{p(\theta)\, f(x|\theta)}{\int p(\theta) f(x|\theta) d\theta}$

Then the posterior likelihood ratio is

$$\frac{p(\theta|x_2)}{p(\theta|x_1)} = \frac{p(\theta) f(x_2|\theta)}{\int p(\theta) f(x_2|\theta) d\theta} \cdot \frac{\int p(\theta) f(x_1|\theta) d\theta}{p(\theta) f(x_1|\theta)}$$

$$= \frac{\int p(\theta) f(x_1|\theta) d\theta}{\int p(\theta) f(x_2|\theta) d\theta} \cdot \frac{f(x_2|\theta)}{f(x_1|\theta)}$$

By definition of MLR, we know for any two values $\theta_1 < \theta_2$ and $T(x_1) < T(x_2)$ that

$$\frac{f(x_1|\theta_2)}{f(x_1|\theta_1)} \leq \frac{f(X_2|\theta_2)}{f(x_2|\theta_1)}$$

and thus $$\frac{f(x_2|\theta_1)}{f(x_1|\theta_1)} \leq \frac{f(x_2|\theta_2)}{f(x_1|\theta_2)} .$$

Since x_1, x_2, θ_1, and θ_2 are all <u>fixed</u> we can write this as

$$\frac{p(\theta_1|x_2)}{p(\theta_1|x_1)} \leq \frac{p(\theta_2|x_2)}{p(\theta_2|x_1)}$$

and conclude that $p(\theta|x)$ has monotone posterior ratio.

In this paper we generally use the same prior distribution for every applicant. This is appropriate when the applicants have the same kind of background. If the applicants are from different groups we might use different prior distribution for the applicants from different groups. The force of this lemma is that with identical prior distributions an ordering on $T(x)$ produces the same ordering as would an investigation of probability ratios from the posterior distribution.

While MLR and MPR are useful conditions, a weaker condition is adequate for our purposes.

Definition 3: A posterior distribution is said to be stochastically increasing (SI) in the statistic T if $T(x_1) \geq T(x_2)$ and for all θ we have $F_{T(x_1)}(\theta) \leq F_{T(x_2)}(\theta)$, where $F_{T(x_1)}(\theta)$ and $F_{T(x_2)}(\theta)$ are the cumulative posterior distributions of θ with observations $T(x_1)$ and $T(x_2)$ respectively.

The theorem that follows is the foundation of this paper. We will use properties of this theorem throughout the paper.

Theorem 1: Let $g(\theta)$ be an arbitrary non-decreasing function of θ, then $E[g(\theta)|T(x)]$ is a non-decreasing function of $T(x)$ if and only if the posterior distribution is stochastically increasing in $T(x)$.

Proof: We first assume that $T(x_1) \geq T(x_2)$ and for all θ we have $F_{T(x_1)}(\theta) \leq F_{T(x_2)}(\theta)$. Proof of sufficiency then follows the argument given by Lehmann (1959, p. 114). We now extend this work and demonstrate necessity.

Assume for any $g(\theta)$, $E[g(\theta)|T(x)]$ is a non-decreasing function of $T(x)$. In particular we assume:

$$g(\theta) = \begin{cases} -1 & \text{if } \theta \leq \theta^* \\ 0 & \text{if } \theta > \theta^*, \end{cases}$$

then $E[g(\theta)|T(x)] = -F_{T(x)}(\theta^*)$.

Thus when $T(x_1) \geq T(x_2)$, we have $-F_{T(x_1)}(\theta) \geq -F_{T(x_2)}(\theta^*)$.

In above θ^* is any $\theta \in \Theta$, thus for any θ and $T(x_1) \geq T(x_2)$ we have $F_{T(x_1)}(\theta) \leq F_{T(x_2)}(\theta)$.

A posterior distribution satisfying MPR also satisfies SI. (Lehmann's proof for MLR requires only trivial modification for this case.) Thus we have the following corollary:

Corollary 1: Suppose $f(x|\theta)$ has MLR with statistic $T(x)$ and $g(\theta)$ is a non-decreasing function of θ, then $E[g(\theta)|T(x)]$ is a non-decreasing function of $T(x)$. (Similarly if $h(\theta)$ is a non-increasing function of θ, then $E[h(\theta) \mid T(x)]$ is a non-increasing function of $T(x)$).

In application $g(\theta)$ will be a utility function $U(\theta)$ and we will refer to this property as the monotone expected utility (MEU) property. The force of this corollary is that a cut-score (CS) x^* can be set is guaranteed. For if θ has MPR and $T(x_2) \leq T(x^*) \leq T(x_1)$,

then $E[g(\theta) \mid T(x_2)] \leq E[g(\theta) \mid T(x^*)] \leq E[g(\theta) \mid T(x_1)]$.

Thus, in summary, we have for an arbitrary utility function

$$\text{MLR} \Rightarrow \text{MPR} \Rightarrow \text{SI} \Leftrightarrow \text{MEU} \Leftarrow \text{CS}$$

Cut Scores for Specific Models

The weakness in the theory of the previous section is that many posterior distributions are <u>not</u> MPR or SI. Specifically in a wide range of applications the posterior marginal distribution will be "t". The following analysis demonstrates that these "t" distributions are not SI or MPR and that the use of a cut-score is not necessarily justified in all cases.

Consider the simple linear regression model,

$$y = \alpha + \beta x + e$$

The value α and β are respectively the intercept and slope of the linear regression function. y is the performance ability and x is the test score which is the same as t in the previous sections. Following the development of Petersen (1976) we assume that the variance of the error is homoscedastic, i.e.,

$$\mathrm{Var}(y \mid x) = \sigma^2$$

for all x. We also assume that e is normally distributed for fixed α, β and x.

Suppose that α, β and $\log \sigma$ are uniformly and independently distributed. Given n pairs of observations (x,y) the posterior predictive distribution y for an applicant with the score $X = x_0$ has a Student t distribution of (n-2) degrees freedom, specifically, (Novick and Jackson, 1974) given $X = x_0$,

$$\frac{y - [\bar{y} + \hat{\beta}\,(x_0 - \bar{x})]}{s[\frac{n+1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x - \bar{x})^2}]^{1/2}} : t(n-2) \tag{1}$$

where $\bar{x} = \Sigma x/n$ $\bar{y} = \Sigma y/n$ $S_x^2 = \Sigma(x - \bar{x}^2)$, $S_y^2 = \Sigma(y - \bar{y})^2$, $S_{xy} = \Sigma(x - \bar{x})\,(y - \bar{y})$, $\hat{\beta} = S_{xy}/S_x^2$ and $s^2 = (S_y^2 - S_{xy}^2/S_x^2)/(n - 2)$.

Let $y^\star$ be the minimum level of satisfactory performance for a threshold utility, then for a given test score $X = x_0$ the expected utility is

$$E[U(y)\,|x_0] = \Pr(Y > y^\star \mid x_0)$$

$$= 1 - \Pr\left\{ t \leq \frac{y^\star - [\bar{y} + \hat{\beta}\,(x_0 - \bar{x})]}{s[\frac{n+1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x - \bar{x})^2}]^{1/2}} \right\} \tag{2}$$

where t is the standard student t variable with (n - 2) degrees of freedom.

To see that the posterior predictive distribution of the student t variable in (1) does not have SI. Let

$$g(x_0) = \frac{y^* - [\bar{y} + \hat{\beta}\,(x_0 - \bar{x})]}{s\left[\frac{n+1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x - \bar{x})^2}\right]^{1/2}}$$

Equation (2) shows that the expected utility increases as $g(x_0)$ decreases. Therefore, the cut-score can be set if $E[U(y) \mid x_0]$ is a decreasing function of x_0. Taking the derivative of $g(x_0)$ we get

$$\frac{\partial}{\partial x_0}\, g(x_0) = \frac{-(y^* - \bar{y})\,\frac{(x_0 - \bar{x})}{\Sigma(x - \bar{x})^2} - \hat{\beta}\,\frac{n+1}{n}}{s\left[\frac{n+1}{n} + \frac{(x_0 - x)^2}{\Sigma(x - \bar{x})^2}\right]^{3/2}}$$

Setting $\frac{\partial}{\partial x_0}\, g(x_0) \leq 0$, we have

$$-(y^* - \bar{y})\,\frac{(x_0 - \bar{x})}{(x - \bar{x})^2} \leq \hat{\beta}\,\frac{n+1}{n}$$

$$\text{or} \quad -(y^* - \bar{y})\,(x_0 - \bar{x}) \leq \frac{n+1}{n}\,\Sigma(x - \bar{x})\,(y - \bar{y}) \tag{3}$$

as the condition required for a cut score. Consider

(i) $y^* = \bar{y}$, then (3) is equivalent to

$$0 \leq \Sigma\,(x - \bar{x})\,(y - \bar{y})$$

In this case a cut score can be set if the corrected sum of cross products (or the slope of the regression line) is non-negative. Now consider

ii) $y^* \neq \bar{y}$ then (3) becomes

$$x_0 \geq \bar{x} - \frac{n+1}{n} \cdot \frac{\Sigma(x-\bar{x})\,(y-\bar{y})}{y^* - \bar{y}} \qquad \text{for } y^* > \bar{y}, \tag{4}$$

and

$$x_0 \leq \bar{x} + \frac{n+1}{n} \cdot \frac{\Sigma(x - \bar{x})\,(y - \bar{y})}{\bar{y} - y^*} \qquad \text{for } y^* < \bar{y} \tag{5}$$

Combining (i) and (ii) for any arbitrary threshold utility y^*, a cut score can be set if

$$\bar{x} - \frac{n+1}{n} \frac{\Sigma(x-\bar{x})(y-\bar{y})}{|y^* - \bar{y}|} \leq x_0 \leq \bar{x} + \frac{n+1}{n} \frac{\Sigma(x-\bar{x})(y-\bar{y})}{|y^* - \bar{y}|} \tag{6}$$

or

$$|x_0 - \bar{x}| \leq \frac{n+1}{n} \frac{\Sigma(x-\bar{x})(y-\bar{y})}{|y^* - \bar{y}|} \tag{7}$$

The equation (7) can be expressed as

$$\frac{|x_0 - \bar{x}|}{s_x} \frac{|y^* - \bar{y}|}{s_y} \leq (n+1)\, r \tag{8}$$

where r is the sample correlation coefficient for (x,y) and

$$s_x = \left(\frac{S_x^2}{n}\right)^{1/2} \quad \text{and} \quad s_y = \left(\frac{S_y^2}{n}\right)^{1/2} .$$

This shows that a cut-score can be set unless n is small, r is small, y^* is distant from $\bar{y}$ and there are x values very distant from $\bar{x}$. This condition may be satisfied for some scores in the case of high selectivity.

The 1968 data on college 7 for junior college test scores from the American College Testing (ACT) Program will be used for the example. The y variable is the first semester grade-point average (GPA) and the predictor variable x is English test score. Since most colleges require a minimum 2.00 for graduation we set $y^* = 2.00$. The original data contain 105 observations. For the purpose of the example we picked the first 10 observations. For $n = 105$, $\bar{x} = 19.13$, $\bar{y} = 2.28$, $S_x^2 = 2766.13$, $S_y^2 = 84.34$, $S_{xy} = 235.47$, $\hat{\beta} = 0.085$, $s = 0.79$ and $D = 849$; for $n = 10$, $\bar{x} = 22.1$, $\bar{y} = 2.66$, $S_x^2 = 140.9$, $S_y^{\ 2} = 6.4$, $S_{xy} = 5.54$, $\hat{\beta} = 0.039$, $s = 0.88$ and $D = 9$,

where $$D = \frac{n+1}{n} \frac{\Sigma(x-\bar{x})(y-\bar{y})}{|y^*-y|}$$

Since the range for ACT English Scores is from 0 to 36, for $n = 105$, the condition in (5) is satisfied for all x_0; therefore, a cut score can be set and the expected utility is an increasing function of the test score. For $n = 10$, the condition in (5) is violated for $x_0 \geq 32$; hence setting a cut score is infeasible. Table 1 contains the values of the expected utility for each sample for $x_0 = 0(2)36$. For $n = 105$, the expected utility is increasing in test score x_0; however, for $n=10$, the expected utility increases and then decreases at $x_0 = 32$. Hence a cut-score cannot be set.

Insert Table 1 about here

It is possible to treat a more general class of utility functions in precisely the same way. Suppose that the utility of selecting a person of ability y is a normal ogive. Let μ_0 and $\sigma_0^{\ 2}$ be the mean and variance for this ogive so that

$$U(y) = \Phi\left(\frac{(y - \mu_0)}{\sigma_0}\right)$$

If n is large the t distribution in (1) is approximated by a normal variable with mean $\bar{y} + \hat{\beta}(x_0 - \bar{x})$ and variance

$$s^2\{[(n+1)/n] + [(x_0 - \bar{x})^2 / \Sigma(x - \bar{x})^2]\}.$$

The expected utility (Novick and Lindley, 1979) of selecting a person with score x_0 is then

$$EU(y) = \Phi\left[\frac{\bar{y} + \hat{\beta}(x_0 - x) - \mu_0}{\left(\sigma_0^2 + s^2\left[\frac{(n+1)}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x - \bar{x})^2}\right]\right)^{1/2}}\right]$$

Following the techniques used previously, the condition for which the cut-score can be set is

$$\frac{|x_0 - \bar{x}|}{s_x} \frac{|\mu_0 - \bar{y}|}{s_y} \leq r(n+1) + r\left(\frac{\sigma_0}{s}\right)^2 \qquad (9)$$

This result may be compared with (8) noting that in the special case $\sigma_0 = 0$ the two expressions agree. For the previous example **(n=10) (9) will be satisfied provided $\sigma_0 > 0.58$. Threshold utility is a special case of** normal ogive utility with $\sigma_0 = 0$ and thus it is seen to be the "worst case". This may generally be true though we offer no proof for other ogival forms.

In a recent topical issue of the Journal of Educational Measurement there was some controversy over the issue of whether or not the setting of cut-scores was feasible. One paper (Glass, 1978) was very pessimistic, another was very optimistic (Hambleton, 1978). Others were less firmly committed on this point. All of the discussions in these papers seemed to be in the context of threshold utility, though it was not always apparent that a clear distinction was being made between the minimum criterion level θ^* in threshold utility and the cut-score x^* in the observation space. Criticism (Glass, 1978) of attempts to specify a threshold cut θ^* seem valid as they are effectively criticisms of threshold utility which Novick and Lindley (1978) clearly suggest is no better than a first rough approximation to a realistic utility function.

The point of the present paper is that cut-scores generally do exist and can be rationally specified to the extent that utility functions can be accurately specified. They do not depend on the assumption of threshold utility. Only the lesser assumptions of monotone likelihood ratio, stochastic increase, or the demonstration of monotone expected utility locally for the particular likelihood, prior distributions and utilities are needed. Thus a focus of work ought to be on the development of coherent bias-free methods of assessing utilities. In this connection a recent discussion by Novick, Chuang, and DeKeyrel (in press) seems relevant The counter argument to this is that the utility of selection often cannot be related to available criteria and that the more important factors needed to be evaluated less formally. This paper does not contribute to the debate on this point but does argue that other problems are manageable.

Selection for a Single Job Category

In this section we consider that there are N candidates applying for one job category A, that has openings. We will study optimal selection procedures for quota free and restricted models.

Suppose job A has openings and let the utility of accepting someone with ability θ be $U_A(\theta)$ and the utility of rejecting someone with ability θ be $U_R(\theta)$. In the case of personnel selections, the greater the ability of the candidate the greater will be his value to potential employers. We may assume that $U_A(\theta)$ is a non-decreasing function of θ and $U_R(\theta)$ is a non-increasing , constant, or more slowly increasing function of θ than $U_A(\theta)$. Then the marginal utility of selection, $U_A(\theta) - U_R(\theta)$, is a non-decreasing function of θ.

<u>Theorem 2</u>: Under the conditions of theorem 1. If $U_A(\theta) - U_R(\theta)$ is a non-decreasing function of θ then there exists a cut-score $T(x^*)$ such that

$$E[U_A(\theta) \mid T(x)] \geq E[U_R(\theta) \mid T(x)] \text{ for } T(x) > T(x^*)$$

and

$$E[U_A(\theta) \mid T(x)] \leq E[U_R(\theta) \mid T(x)] \text{ for } T(x) < T(x^*)$$

In extreme cases $T(x^*)$ may equal $-\infty$ or $+\infty$.

The above result is a special case of a theorem in Karlin and Rubin (1956). The proofs can be found in Lehmann (1959, p. 74). The importance of this theorem is that the existence of cut-score is derived rather than assumed. Thus in the case of monotone posterior ratio and monotone utility functions a cut score can always be used to maximize utility.

Theorem 2 shows that if the applicant's test score T(x) is greater then $T(x^*)$ then the expected utility for acceptance is higher than rejection (hence he should be accepted). This is equivalent to saying that the expectation of $U_A(\theta) - U_R(\theta)$ is non-negative, that is,

$$E[U_A(\theta) - U_R(\theta) \mid T(x)] \geq 0 \quad \text{for } T(x) > T(x^*).$$

Similarly, we have

$$E[U_A(\theta) - U_R(\theta) \mid T(x)] \leq 0 \quad \text{for } T(x) < T(x^*).$$

Therefore, the cut score $T(x^*)$ is the point at which $E[U_A(\theta) - U_R(\theta) \mid T(x)]$ changes sign; that is, $E[U_A(\theta) - U_R(\theta) \mid T(x)]$ changes from nonpositive values to nonnegative values at $T(x^*)$. In particular, if $E[U_A(\theta) - U_R(\theta) \mid T(x)]$ is continuous at $T(x^*)$ then

$$E[U_A(\theta) - U_R(\theta) \mid T(x^*)] = 0.$$

The following theorem shows that under our conditions, if we have two applicants and exactly one opening we should accept the one with the higher test score.

Theorem 3: Under the conditions of the theorem 1. If $U_A(\theta) - U_R(\theta)$ is a non-decreasing function of θ and $T(x_i) > T(x_j)$, then

$$V_A(T(x_i)) + V_R((x_j)) \geq V_A(T(x_j)) + V_R(T(x_i)),$$

where $V_A(T(x)) \equiv E[U_A(\theta) \mid T(x)]$ and $V_R(T(x)) \equiv E[U_R(\theta) \mid T(x)]$

Proof: By Theorem 1, $E[U_A(\theta) - U_R(\theta) \mid T(x)]$ is a non-decreasing function of T(x). Thus,

$$E[U_A(\theta) - U_R(\theta) \mid T(x_i)] \geq E[U_A(\theta) - U_R(\theta) \mid T(x_j)]$$

so

$$V_A(T(x_i)) - V_R(T(x_i)) \geq V_A(T(x_j)) - V_R(T(x_j)),$$

then

$$V_A(T(x_i)) + V_R(T(x_j)) \geq V_A(T(x_j)) + V_R(T(x_i)).$$

Assume that the test scores of N applicants are $T(x_1)$, $T(x_2)$, ...

$T(x_N)$, without loss of generality we can assume $T(x_1) \geq T(x_2)$, ... $\geq T(x_N)$. For job category A we may have three situations:

(1) quota free, i.e., we may accept as many applicants as we want;

(2) we want to accept <u>exactly</u> N_A applicants where $N_A \leq N$;

(3) we want to accept <u>at most</u> N_A applicants where $N_A \leq N$.

Note that if $N_A = N$ then (3) specializes to (1). The decision rules for these cases are very simple. From Theorem 2 there exists a cut-score, say C, such that

$$V_A(T(x)) \geq V_R(T(x)) \quad \text{for } T(x) > C$$

and

$$V_A(T(x)) \leq V_R(T(x)) \text{ for } T(x) < C$$

then

(a) The optimal decision procedure for situation (1) is to accept the i-th applicant if $T(x_i) > C$ and reject him if $T(x_i) < C$.

(b) The optimal decision procedure for situation (2) is to accept those N_A applicants who have the highest test scores (i.e., accept those who have scores $T(x_1)$, $T(x_2)$ ... $T(x_{N_A})$).

(c) The optimal decision procedure for situation (3) is: if $T(x_{N_A}) \geq C$ then accept the first N_A applicants, i.e., the same as (b); if $T(x_{N_A}) < C$ then we will accept applicants whose scores are greater than C, i.e., the same as in (a).

<u>Proof</u>:

(a) is trivial, because we maximize our utility of assigning each applicant;thus we have maximum total utility.

(b) is a direct extension of Theorem 3. If we exchange our decision about any two applicants, we can see by Theorem 3 our total expected utility will not increase. Thus we have optimal decision procedure.

(c) is a combination of (a) and (b) and we can see the procedure is optimal immediately.

Again we emphasize that this theorem is generally false in the absence of SI but the use of a cut-score may be valid with reasonable practice. We also note that while we assume $U_A(\theta) - U_R(\theta)$ is nondecreasing in θ we are not assuming that the derivative $U_A'(\theta)$ is nondecreasing in θ. In particular the following schematic is valid and should be considered typical.

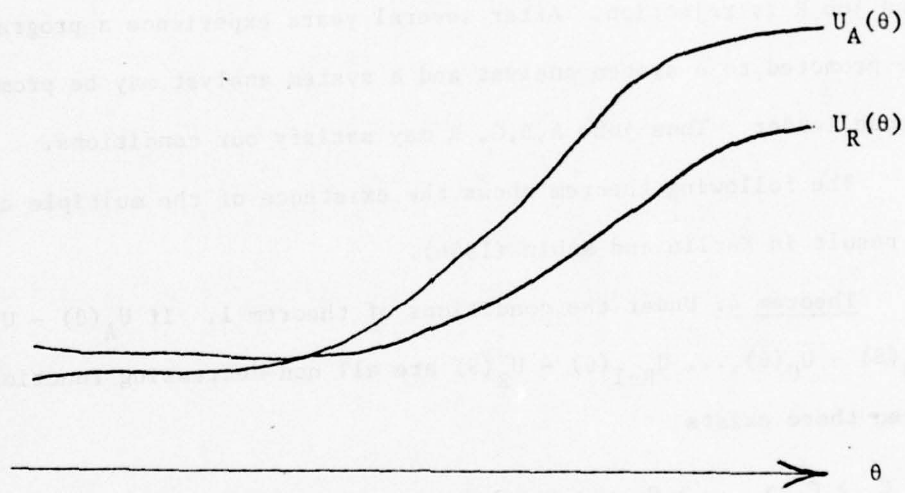


Figure 1 $U_A(\theta) - U_R(\theta)$

Selection for Multiple Job Categories

In this part we consider that jobs A, B, C, ... R have openings. We also assume there are N applicants and the utility of assigning someone with ability θ to jobs A, B, C, ... R are $U_A(\theta)$, $U_B(\theta)$, ... $U_R(\theta)$ respectively. In general these kinds of problems are very difficult. We will consider an important special case of the situation by assuming $U_A'(\theta) \geq U_B'(\theta) \ldots \geq U_R'(\theta)$ for all θ. (We can weaken this condition by assuming $U_A(\theta) - U_B(\theta)$, $U_B(\theta) - U_C(\theta)$, ... $U_{R-1}(\theta) - U_R(\theta)$ are all non-decreasing functions of θ.) An example of this special case may be that job A is group leader, job B is system analyst, job C is programmer and job R is rejection. After several years experience a programmer may be promoted to a system analyst and a system analyst may be promoted to a group leader. Thus jobs A,B,C, R may satisfy our conditions.

The following theorem shows the existence of the multiple cut-scores, a result in Karlin and Rubin (1956).

Theorem 4. Under the conditions of theorem 1. If $U_A(\theta) - U_B(\theta)$, $U_B(\theta) - U_C(\theta)$,... $U_{R-1}(\theta) - U_R(\theta)$ are all non-decreasing functions of θ, then there exists

$C_A \geq C_B \geq \ldots \geq C_{R-1}$ such that

$$
\begin{aligned}
&\text{if} \quad T(x) > C_A \text{ then } V_A(T(x)) = V_* \,(T(x)) \\
&\text{if } C_A > T(x) > C_B \text{ then } V_B(T(x)) = V_* \,(T(x)) \\
&\text{if } C_B > T(x) > C_C \text{ then } V_C(T(x)) = V_* \,(T(x)) \\
&\quad\vdots \\
&\text{if } C_{R-1} \geq T(x) \quad \text{ then } V_R\,(T(x)) = V_*(T(x))
\end{aligned}
\tag{10}
$$

where $V_* \,(T(x)) = \text{Max}\left\{V_A(T(x)),\ V_B(T(x)),\ \ldots,\ V_R(T(x))\right\}$.

From the above result we can see very clearly that if we have an applicant, then we use C_A, C_B,... C_{R-1} as cut-scores and decide to which job to assign him. For example if $T(x) > C_A$ then we assign the applicant to job A, if $C_A > T(x) > C_B$ then we assign the applicant to job B ... etc. Intuitively we assign a person with higher score to a job that gives higher marginal utility (for example, job A). Generally we find that C_A, C_B,... C_{R-1} are distinct. Thus every job has a chance to receive the applicant. The following theorem shows how to find $C_A, C_B, \ldots, C_{R-1}$. Its proof can be obtained by repeatedly applying theorem 1.

<u>Theorem 5</u>: Under the conditions of theorem 4. Assume $E[U_A(\theta) - U_B(\theta) \mid T(x)]$, $E[U_B(\theta) - U_C(\theta) \mid T(x)], \ldots, E[U_{R-1}(\theta) - U_R(\theta) \mid T(x)]$ are continuous functions of $T(x)$, let

$$
\begin{aligned}
&E[U_A(\theta) - U_B(\theta) \mid T(x) = C_A] = 0 \\
&E[U_B(\theta) - U_C(\theta) \mid T(x) = C_B] = 0 \\
&\qquad \vdots \\
&E[U_{R-1}(\theta) - U_R(\theta) \mid T(x) = C_{R-1}] = 0
\end{aligned}
\tag{11}
$$

If $C_A \geq C_B \geq, \ldots C_{R-1}$, then the equation (10) holds. Hence, $C_A, C_B, \ldots, C_{R-1}$ are the cut-scores.

For the conditions $C_A \geq C_B \geq \ldots \geq C_{R-1}$ in theorem 5 , if one of the equalities is violated, then the corresponding scores for the solutions in (11) are not the cut-scores. Suppose that there are only three jobs, A, B, and C. Let C_A and C_B be the solutions of (11) such that $C_B \geq C_A$, and let

$$E[U_A(\theta) - U_C(\theta) \mid T(x) = C_{AC}] = 0$$

$$\text{Then } C_B \geq C_{AC} \geq C_A \text{ and} \tag{12}$$

$$
\begin{aligned}
&\text{if } T(x) \geq C_{AC} \quad \text{then } V_A(T(x)) = V_*(T(x)) \\
&\text{if } T(x) \leq C_{AC} \quad \text{then } V_C(T(x)) = V_*(T(x)).
\end{aligned}
$$

Thus, C_{AC} of the equation (12) is the only cut-score, and job B does not have a chance to receive any applicant.

The condition $U'_A(\theta) \geq U'_B(\theta) \geq \ldots \geq U'_R(\theta)$ or the alternative $U_A(\theta) - U_B(\theta)$, $U_B(\theta) - U_C(\theta)$, ... $U_{R-1}(\theta) - U_R(\theta)$ are all non-decreasing functions of θ can be relaxed to include important additional cases. Suppose that for $\theta > \theta_I$, $U_A(\theta) - U_B(\theta)$ is a positive function of θ; for $\theta_I > \theta > \theta_{II}$, $U_B(\theta) - U_C(\theta)$ is a positive function of θ,etc. Suppose further that a multiple cut-score solution yields cut-scores C_A, and C_B, then it is evident that the computed solution is a valid solution. Thus we see that the required monotonicity properties are local rather than global properties.

The monotonicity assumption on utility differences in the weaker form considered above seems eminently reasonable in both employment and educational applications. Consider an underlying "general ability, θ" and the occupational careers of physicist (A), electrical engineer (B), and electrician (C). Then the utility structure pictured in figure 2 may be valid.

Insert Figure 2 About Here

The following example demonstrates the use of cut-scores in multiple job assignments. Suppose there are three jobs. A, B and C are available and suppose that the utilities of jobs A, B, and C are

$$U_A(y) = \Phi\left(\frac{y-2.6}{1}\right)$$

$$U_B(y) = \Phi\left(\frac{y-2.2}{2}\right)$$

and

$$U_C(y) = \Phi\left(\frac{y-2}{3}\right)$$

Using the 1968 data on college 7 of ACT scores given in section 2, the expected utilities for each job are given in column 2 to column 4 of Table 2. Column 5 is the difference of column 2 and column 3, and column 6 is the difference of column 3 and column 4. Column 5 and column 6 change sign at x = 30 and x = 24, respectively. The optimal assignment is

If $x \geq 30$ assign applicant to job A.

If $30 > x \geq 24$ assign applicant to job B.

If $24 > x$ assign applicant to job C.

Insert Table 2 Near Here

Again consider N applicants and without loss of generality assume their test score satisfies $T(x_1) \geq T(x_2) \geq \ldots \geq T(x_N)$. We may have the following situations:

(1) quota free for all jobs, i.e. we can accept as many applicants as we want for each job.

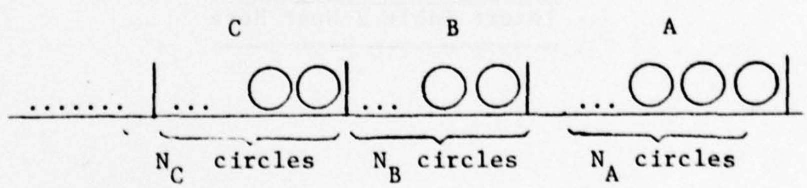
(2) We want to accept exactly $N_A, N_B, N_C \ldots N_R$ applicants for jobs A,B,C,...R respectively. So in this case we assume $N_A+N_B+N_C+\ldots+N_R = N$.

(3) For every job A,B,C,...R we want to accept at most N_A, N_B, ... N_R respectively. So in this case we assume $N_A+N_B+N_C+\ldots+N_R \geq N$.

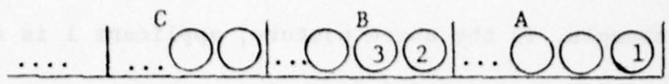
Before we find optimal decision procedure for the above situations we should notice that (3) is the most general one. In situation (3) if we have the additional condition that $N_A=N_B=N_C \ldots = N_R = N$ then (3) specializes to (1). If $N_A+N_B+\ldots+N_R = N$ then we can also see (3) specializes to (2). In the following we will consider situation (3) first and discuss situations (1) and (2) in the corollary at the end of this section.

Before we develop an optimal decision procedure for situation (3) we need some definitions and preliminary results.

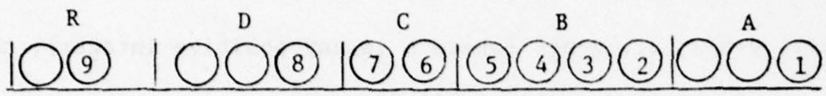
We can illustrate situation (3) as follows



where A,B,C... are job categories, and the number of circles under each job is the number of openings for that job category. When we assign an applicant to a job we put his number in the corresponding circle. For example, the following picture represents our assigning applicant 1 to job A, applicants 2 and 3 to job B.



By theorem (3) of the previous section and the assumption that $T(x_1) \geq T(x_2) \ldots \geq T(x_N)$, we can see that an applicant with a higher score should occupy a position to the right of a lower score applicant. Thus, we will only consider assignments that satisfy this condition. The following assignment satisfies this condition.



Here we have 9 applicants and $N_A = 3$, $N_B = 4$, $N_C = 2$, $N_D = 3$ and $N_R = 2$.

We can also put index on the openings in the way that openings in job A have index $1,2,\ldots N_A$ and openings for job B have index $N_A+1,\ldots N_A+N_B$ respectively.

After assignment of all applicants we need the following definition.

Definition Chain is a set of consecutive applicants such that the highest scoring applicant in the set occupies the first opening of the job he got. All the job openings between the one occupied by the highest score and the one occupied by the lowest score (in the chain) are all occupied by the members of the chain. And there are still opening(s) for the job that the lowest scored applicant in the chain got. Also, there are still opening(s) for the job just to the right of the opening(s) the highest scoring applicant got, if there are any.

For example, in the above picture, applicant 1 is a chain and the set of applicants 2,3,4,5,6,7,8 is also a chain.

From the above definition, we can have the following lemma.

Lemma 2: Suppose we have the optimal assignment of applicants 1,2,...N. Let the highest scoring applicant in a chain be $i+1^{th}$ applicant. We assume there are K applicants in the chain occupying v+1, v+2,...v+k indexed position. If we move j+1, j+2,...j+ℓ (where $i+1 \leq j+1 \leq j+\ell \leq i+k$, i.e. all are in the chain) from positions v+1, v+2, ... v+ℓ to positions v-n+1, v-n+2, ... v-n+ℓ (where n is any positive integer), then the the total expected utility will decrease.

Proof: We first show that we shift i+1, i+2, ... i+ℓ applicants from v+1, v+2, ...v+ℓ to v+n+1, v+n+2, ... v+n+ℓ then the total expected utility will decrease or not change.

When i+1, i+2,...i+ℓ applicants shift from v+1, v+2,...v+ℓ to v, v+1,...v+ℓ-1 we can see the total expected utility will reduce or not change because before the shift we already have optimal assignment.

Now i+2, i+3,...i+ℓ occupy positions v+1, v+2,...v+ℓ-1, respectively. By theorem 2 of the previous section if we shift applicant i+1, i+2,...i+ℓ-1 from positions v+1, v+2,...v+ℓ-1 to v,v+1,...v+ℓ-2 the total expected value will decrease or not change. We can see that shifting applicants i+2, i+3,...i+ℓ from v+1, v+2,...v+ℓ-1 to v, v+1,...v+ℓ-2 will reduce or not change total expected utility.

For those who occupy positions (note we only consider applicants i+1, i+2,...i+k) less than v+1 we don't consider them because the more to the right the less expected utility we will get from them.

Similarly, if we move $i+3, i+4, \ldots i+\ell$ from $v+1, v+2, \ldots v+\ell-2$ openings one position to the right the total expected utility will not increase. Thus, we can see that moving $i+1, i+2, \ldots i+\ell$ from positions $v+1, v+2, \ldots v+\ell$ to $v+1-n, v+2-n, \ldots v+\ell-n$ will not increase our total expected utility.

Next we consider moving $j+1, j+2, \ldots j+\ell$ applicants from $v+1, v+2, \ldots v+\ell$ to positions $v-n+1, v-n+2, \ldots v-n+\ell$. By the above result and Theorem 2 of the previous section we can see this action will not increase the total expected utility.

Now we are ready to show optimal decision procedure for situation (3). The procedure is as follows:

We arrange the applicants according to their test scores and start from the first person (who has the highest score) then the second, third, etc... until the one who has the lowest test score using the following assigning algorithm. We consider ith applicant with test score $T(x_i)$ $(i=1,2,\ldots N)$.

(i) assign the applicant to the job that gives highest expected utility.

(ii) if the job in (i) is filled (i.e., filled by some people who have higher test scores than $T(x_i)$), then assign i to the same job as (i-1)th applicant.

(iii) if the job in (ii) is also filled we have to consider two possible ways: (1) assign i to the next job opening that has lower marginal utility than the job (i-1) applicant was assigned or (2) assign i to the same job as (i-1) applicant, but now this job accepts more applicants than its limit. Thus, we shift all applicants in the chain that contains (i-1)th applicant one position to the right. From these two ways we select the one that has higher expected utility.

The above decision procedure is optimal since:

When (i) is true we can see immediately we have assigned applicant 1,2,... i-1, i optimally because we assigned applicant 1,2,...i-1 and i optimally.

To consider (ii) we have to compare the assignment that satisfies the condition that an applicant with a higher score should occupy a position on the right. (This is another way of expressing Theorem 2 in the previous section).

We know applicant i has to occupy an opening at least to the right of the opening we originally assigned, or the new assignment will not increase our total expected utility.

Let the new arrangement satisfy the above conditions. We will shift our original arrangement in steps to the new arrangement such that each step our total expected utility will not increase. And thus our originally assignment has highest expected utility.

We first shift the last chain in our original arrangement one position to the right. By (2) in Lemma 2, our total expected utility will not increase (note ith applicant does not change job, although he changes openings). Now we shift this chain to the right until ith applicant occupies the same position as a new arrangement. Every shift, by (3) in Lemma 2, will not increase total expected utility. We now shift all except i to the right until i-lth applicant occupies the position in the new arrangement. We now move applicants in the last chain, except i-1 and i to the right until i-2th applicant occupies the positions in the new arrangement and each step will not increase our total expected utility, thus (ii) gets the highest expected utility.

It can be shown that (iii) is optimal by exactly the same way as in (ii).

Corollary:

(1) For a quota free situation, the optimal decision procedure is to assign each applicant to the job that gives highest expected utility.

(2) For situation (2), the optimal decision procedure is to assign applicant $1,2,\ldots N_A$ to job A, N_A+1, N_A+2, ... N_A+N_B to job B,..., and the last N_R applicants to job R.

The above corollary is a direct result of our previous decision procedure.

Selections Considering Applicant's Preference

In this part, we take applicant's preference into consideration. Suppose we have two jobs and N applicants. Every applicant states which job he prefers. Let these two jobs be A and B and each has n_A and n_B openings. We further assume $n_A + n_B \geq N$ and the decision maker know each applicant's preference between jobs A and B. A decision maker assigns each applicant to either job A or B but not both

Let $U_1(\theta)$ and $U_2(\theta)$ satisfy:

$$U_1(\theta) \geq U_2(\theta) \qquad (2)$$

$$U_1'(\theta) \geq U_2'(\theta) \qquad (3)$$

Now we define utility functions. Suppose ith applicant prefers job A to B. We assume

$$U_{iA}(\theta) = U_1(\theta)$$

$$U_{iB}(\theta) = U_2(\theta)$$

where $U_{iA}(\theta)$ and $U_{iB}(\theta)$ are the utility function if we assign the ith applicant to jobs A and B respectively.

Similarly, if jth applicant prefers job B to A, then

$$U_{jA}(\theta) = U_2(\theta)$$

$$U_{jB}(\theta) = U_1(\theta)$$

Every applicant will have higher utility if we assign him to the job be perferred. Thus assumption (2) is very reasonable. We will see the case when (3) is not true later in this part.

A trivial special case is when $n_A = n_B = N$, then we assign every applicant to the job he prefers.

Under the assumptions, the decision procedure that will maximize the total sum of expected utility is as follows:

Arrange the applicants according to $T(x_1) \geq T(x_2) \ldots \geq T(x_n)$. Starting from the first to the last applicant, assign each (say ith) applicant to the job he preferred, if there is no opening for that job (say already filled by 1, 2, 3 ... i - 1 applicant) then assign him to the other job.

Next we prove the above algorithm maximizes the total expected utility. Suppose at the end job A still has openings, then we cannot move any applicants from job B to A. Those in job B prefer job B to A, (or they will be in job A), thus moving them to A will not increase our total expected utility. When job B still has openings we can prove optimality in the same way.

Next suppose we assign j to job A and k to job B, we show exchange of jobs between j and k will not increase our total expected utility. Without loss of generality we assume $T(x_j) > T(x_k)$, and thus j applicant prefers job A to job B. If k applicant prefers job B to job A, then j and k are happy and we should not exchange their jobs.

If k applicant also prefers job A to B then $U_{jA}(\theta) = U_{kA}(\theta) = U_1(\theta)$ and $U_B\ (\theta) = U_{kB}(\theta) = U_2(\theta)$. By Theorem 2, we have

$$V_A(T(x_j)) + V_B(T(x_k)) \geq V_A(T(x_k)) + V_B(T(x_j))$$

Thus exchange j and k's job will not increase total expected utility. Thus we have optimal decision procedure.

If we assume $U_1(\theta)$ and $U_2(\theta)$ satisfy:

$$U_1(\theta) \geq U_2(\theta) \qquad (2)$$

$$U_2'(\theta) \geq U_1'(\theta) \qquad (3)'$$

and when i prefers job A to B then $U_{iA}(\theta) = U_1(\theta)$ and $U_{iB}(\theta) = U_2(\theta)$.

Then the optimal decision procedure is as follows: Arrange $T(x_1) \geq T(x_2) \ldots \geq T(x_n)$, but now starting from Nth applicant (the one who has lowest test score) till the first applicant, assign any applicant to the job he preferred, if no opening there then assign him to the other job. The proof is the same as previous case.

In this case we honored lower score applicants first. This may not be acceptable in most of the real world cases.

Another situation is that $U_1'(\theta) = U_2'(\theta)$, i.e. $U_1(\theta)$ and $U_2(\theta)$ differ by a constant. Optimal decision procedure is started from any one applicant (each time arbitrary choose next applicant), if the job he preferred still has opening assign him to there if no opening, assign him to the other job. We can see in this case a lot of arrangements have maximum expected utility.

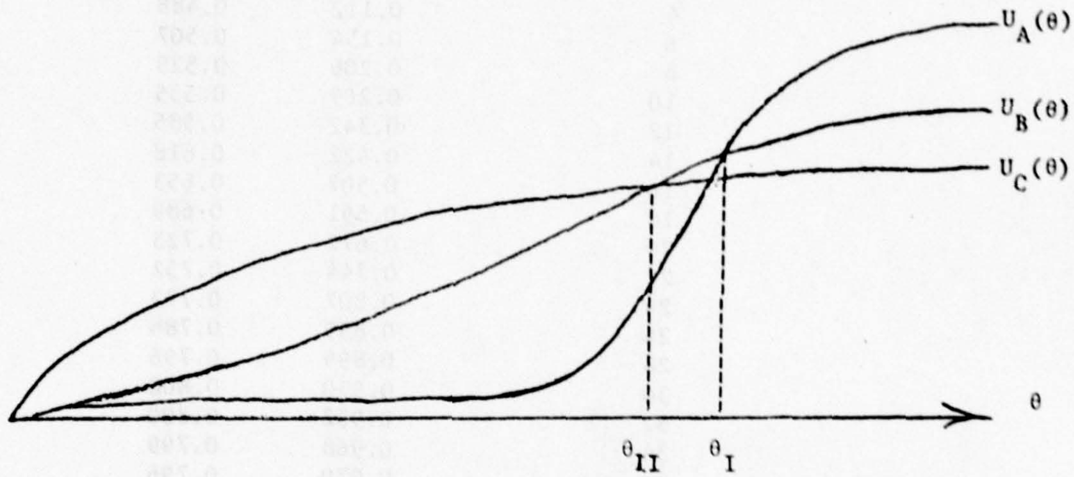


Figure 2

Table 1. The expected utilities for selection for two sample sizes at the various points of the test scores.

	Expected Utilities	
Score	n = 105	n = 10
0	0.057	0.457
2	0.081	0.471
4	0.112	0.488
6	0.154	0.507
8	0.206	0.529
10	0.269	0.555
12	0.342	0.585
14	0.422	0.618
16	0.507	0.653
18	0.591	0.689
20	0.672	0.723
22	0.744	0.752
24	0.807	0.773
26	0.859	0.788
28	0.899	0.796
30	0.930	0.800
32	0.952	0.800
34	0.968	0.799
36	0.979	0.796

Table 2. The expected utilities and the differences of three normal utilities

	Expected Utilities			Difference	
Score	U_A (y)	U_B (y)	U_C (y)	U_A (y) - U_B (y)	U_B (y) - U_C (y)
0	0.070	0.239	0.333	-0.169	-0.094
2	0.088	0.263	0.353	-0.176	-0.090
4	0.109	0.289	0.373	-0.180	-0.084
6	0.134	0.316	0.394	-0.182	-0.078
8	0.164	0.344	0.415	-0.181	-0.071
10	0.197	0.374	0.437	-0.176	-0.063
12	0.236	0.404	0.458	-0.168	-0.055
14	0.278	0.434	0.480	-0.157	-0.046
16	0.324	0.466	0.502	-0.142	-0.036
18	0.373	0.498	0.524	-0.125	-0.026
20	0.424	0.528	0.545	-0.105	-0.017
22	0.476	0.560	0.567	-0.083	-0.007
24	0.529	0.591	0.588	-0.061	0.002
26	0.582	0.621	0.609	-0.039	0.011
28	0.632	0.650	0.630	-0.018	0.020
30	0.680	0.679	0.651	0.002	0.028
32	0.725	0.706	0.670	0.019	0.036
34	0.766	0.732	0.690	0.034	0.043
36	0.803	0.757	0.709	0.046	0.049

References

DeGroot, M. H. Optimal statistical decisions. New York: McGraw-Hill, 1970.

Glass, Gene V. Standards and criteria. Journal of Educational Measurement, 15, 237-261, 1978.

Gross, A. L., & Su, W. H. Defining a "fair" or "unbiased" selection model: A question of utilities. Journal of Applied Psychology, 60, 345-351, 1975.

Hambleton, Ronald K. On the use of cut-off scores with criterion-referenced tests in instructional settings. Journal of Educational Measurement, 15, 277-290, 1978.

Karlin, S. Polya type distributions II. Ann. Math. Statist., 28, 281-308, 1957a.

Karlin, S. Polya type distributions, III. Admissibility for multi-action problems. Ann. Math. Statist., 28, 839-860, 1957b.

Karlin, S. and Rubin, H. The Theory of decision procedures for distributions with monotone likelihood ratio. Ann. Math. Statist., 27, 272-299, 1956.

Lehmann, E. L. Testing Statistical Hypothesis. New York: John Wiley & Sons, Inc., 1959.

Novick, M. R., Chuang, D., & DeKeyrel, D. Local and regional coherence utility assessment procedures. Trabajos de Estadistica (in press).

Novick, M. R. & Jackson, P. H. Statistical methods for educational and psychological research. New York: McGraw-Hill, 1974.

Novick, M. R. & Lindley, D. V. The use of more realistic utility functions in educational applications. Journal of Educational Measurement, 3, 181-192, 1978.

Novick, M. R. & Lindley, D. V. Fixed state assessment of utility functions. Journal of the American Statistical Association, 74, 306-311, 1979.

Novick, M. R. & Petersen, N. S. Towards equalizing educational and employment opportunity. Journal of Educational Measurement, 13, 77-88, 1976.

Petersen, N. S. An expected utility model for "optimal" selection. Journal of Educational Statistics, 4, 333-358, 1976.

Petersen, N. S. & Novick, M. R. An evaluation of some models for culture-fair selection. Journal of Educational Measurement, 13, 3-31, 1976.

Van der Linden, W. J. & Mellenbergh, G. J. Optimal cutting scores using a linear loss function. Applied Psychological Measurement, 1, 593-599, 1977.

Navy

1 Dr. Ed Aiken
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Jack R. Borsting
Provost & Academic Dean
U.S. Naval Postgraduate School
Monterey, CA 93940

1 Dr. Robert Breaux
Code N-71
NAVTRAEQUIPCEN
Orlando, FL 32813

1 MR. MAURICE CALLAHAN
Pers 23a
Bureau of Naval Personnel
Washington, DC 20370

1 Dr. Richard Elster
Department of Administrative Sciences
Naval Postgraduate School
Monterey, CA 93940

1 DR. PAT FEDERICO
NAVY PERSONNEL R&D CENTER
SAN DIEGO, CA 92152

1 Dr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. John Ford
Navy Personnel R&D Center
San Diego, CA 92152

1 CAPT. D.M. GRAGG, MC, USN
HEAD, SECTION ON MEDICAL EDUCATION
UNIFORMED SERVICES UNIV. OF THE
HEALTH SCIENCES
6917 ARLINGTON ROAD
BETHESDA, MD 20014

1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054

Navy

1 Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

1 CHAIRMAN, LEADERSHIP & LAW DEPT.
DIV. OF PROFESSIONAL DEVELOPMMENT
U.S. NAVAL ACADEMYY
ANNAPOLIS, MD 21402

1 Dr. William L. Maloy
Principal Civilian Advisor for
Education and Training
Naval Training Command, Code 00A
Pensacola, FL 32508

1 CAPT Richard L. Martin
USS Francis Marion (LPA-249)
FPO New York, NY 09501

1 Dr. James McBride
Code 301
Navy Personnel R&D Center
San Diego, CA 92152

1 DR. WILLIAM MONTAGUE
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 Library
Navy Personnel R&D Center
San Diego, CA 92152

6 Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

1 OFFICE OF CIVILIAN PERSONNEL
(CODE 26)
DEPT. OF THE NAVY
WASHINGTON, DC 20390

1 JOHN OLSEN
CHIEF OF NAVAL EDUCATION &
TRAINING SUPPORT
PENSACOLA, FL 32509

Navy

1 Psychologist
ONR Branch Office
495 Summer Street
Boston, MA 02210

1 Psychologist
ONR Branch Office
536 S. Clark Street
Chicago, IL 60605

1 Office of Naval Research
Code 200
Arlington, VA 22217

1 Code 436
Office of Naval Research
Arlington, VA 22217

1 Office of Naval Research
Code 437
800 N. Quincy SStreet
Arlington, VA 22217

5 Personnel & Training Research Programs
(Code 458)
Office of Naval Research
Arlington, VA 22217

1 Psychologist
OFFICE OF NAVAL RESEARCH BRANCH
223 OLD MARYLEBONE ROAD
LONDON, NW, 15TH ENGLAND

1 Psychologist
ONR Branch Office
1030 East Green Street
Pasadena, CA 91101

1 Scientific Director
Office of Naval Research
Scientific Liaison Group/Tokyo
American Embassy
APO San Francisco, CA 96503

1 Office of the Chief of Naval Operations
Research, Development, and Studies Branc
(OP-102)
Washington, DC 20350

Navy

1 Scientific Advisor to the Chief of
Naval Personnel (Pers-Or)
Naval Bureau of Personnel
Room 4410, Arlington Annex
Washington, DC 20370

1 LT Frank C. Petho, MSC, USNR (Ph.D)
Code L51
Naval Aerospace Medical Research Laborat
Pensacola, FL 32508

1 DR. RICHARD A. POLLAK
ACADEMIC COMPUTING CENTER
U.S. NAVAL ACADEMY
ANNAPOLIS, MD 21402

1 Mr. Arnold Rubenstein
Naval Personnel Support Technology
Naval Material Command (08T244)
Room 1044, Crystal Plaza #5
2221 Jefferson Davis Highway
Arlington, VA 20360

1 A. A. SJOHOLM
TECH. SUPPORT, CODE 201
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Mr. Robert Smith
Office of Chief of Naval Operations
OP-987E
Washington, DC 20350

1 Dr. Alfred F. Smode
Training Analysis & Evaluation Group
(TAEG)
Dept. of the Navy
Orlando, FL 32813

1 Dr. Richard Sorensen
Navy Personnel R&D Center
San Diego, CA 92152

1 CDR Charles J. Theisen, JR. MSC, USN
Head Human Factors Engineering Div.
Naval Air Development Center
Warminster, PA 18974

Navy

1 W. Gary Thomson
Naval Ocean Systems Center
Code 7132
San Diego, CA 92152

1 Dr. Ronald Weitzman
Department of Administrative Sciences
U. S. Naval Postgraduate School
Monterey, CA 93940

1 DR. MARTIN F. WISKOFF
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

Army

1 Technical Director
U. S. Army Research Institute for the Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 HQ USAREUE & 7th Army
CDCSOPS
USAAREUE Director of GED
APO New York 09403

1 DR. RALPH DUSEK
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 Dr. Myron Fischl
U.S. Army Research Institute for the Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Beatrice J. Farr
Army Research Institute (PERI-OK)
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Milt Maier
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 Dr. Harold F. O'Neil, Jr.
ATTN: PERI-OK
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 Dr. Robert Ross
U.S. Army Research Institute for the Social and Behavioral Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Robert Sasmor
U. S. Army Research Institute for the Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

Army

1 Director, Training Development
U.S. Army Administration Center
ATTN: Dr. Sherrill
Ft. Benjamin Harrison, IN 46218

1 Dr. Frederick Steinheiser
U. S. Army Reserch Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Joseph Ward
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Air Force

1 Air Force Human Resources Lab
AFHRL/PED
Brooks AFB, TX 78235

1 Air University Library
AUL/LSE 76/443
Maxwell AFB, AL 36112

1 Dr. Philip De Leo
AFHRL/TT
Lowry AFB, CO 80230

1 DR. G. A. ECKSTRAND
AFHRL/AS
WRIGHT-PATTERSON AFB, OH 45433

1 Dr. Genevieve Haddad
Program Manager
Life Sciences Directorate
AFOSR
Bolling AFB, DC 20332

1 CDR. MERCER
CNET LIAISON OFFICER
AFHRL/FLYING TRAINING DIV.
WILLIAMS AFB, AZ 85224

1 Dr. Ross L. Morgan (AFHRL/ASR)
Wright -Patterson AFB
Ohio 45433

1 Dr. Roger Pennell
AFHRL/TT
Lowry AFB, CO 80230

1 Personnel Analysis Division
HQ USAF/DPXXA
Washington, DC 20330

1 Research Branch
AFMPC/DPMYP
Randolph AFB, TX 78148

1 Dr. Malcolm Ree
AFHRL/PED
Brooks AFB, TX 78235

Air Force

1 Dr. Marty Rockway (AFHRL/TT)
Lowry AFB
Colorado 80230

1 Jack A. Thorpe, Capt, USAF
Program Manager
Life Sciences Directorate
AFOSR
Bolling AFB, DC 20332

1 Brian K. Waters, LCOL, USAF
Air University
Maxwell AFB
Montgomery, AL 36112

Marines

1 H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134

1 Director, Office of Manpower Utilization
HQ, Marine Corps (MPU)
BCB, Bldg. 2009
Quantico, VA 22134

1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

CoastGuard

1 Mr. Richard Lanterman
PSYCHOLOGICAL RESEARCH (G-P-1/62)
U.S. COAST GUARD HQ
WASHINGTON, DC 20590

1 Dr. Thomas Warm
U. S. Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Other DoD

12 Defense Documentation Center
Cameron Station, Bldg. 5
Alexandria, VA 22314
Attn: TC

1 Dr. Dexter Fletcher
ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BLVD.
ARLINGTON, VA 22209

1 Dr. William Graham
Testing Directorate
MEPCOM
Ft. Sheridan, IL 60037

1 Military Assistant for Training and
Personnel Technology
Office of the Under Secretary of Defense
for Research & Engineering
Room 3D129, The Pentagon
Washington, DC 20301

1 MAJOR Wayne Sellman, USAF
Office of the Assistant Secretary
of Defense (MRA&L)
3B930 The Pentagon
Washington, DC 20301

Civil Govt

1 Dr. Susan Chipman
Basic Skills Program
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. William Gorham, Director
Personnel R&D Center
Office of Personnel Managment
1900 E Street NW
Washington, DC 20415

1 Dr. Joseph I. Lipson
Division of Science Education
Room W-638
National Science Foundation
Washington, DC 20550

1 Dr. John Mays
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Arthur Melmed
National Intitute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Andrew R. Molnar
Science Education Dev.
and Research
National Science Foundation
Washington, DC 20550

1 Dr. Lalitha P. Sanathanan
Environmental Impact Studies Division
Argonne National Laboratory
9700 S. Cass Avenue
Argonne, IL 60439

1 Dr. Jeffrey Schiller
National Institute of Education
1200 19th St. NW
Washington, DC 20208

Civil Govt

1 Dr. Thomas G. Sticht
Basic Skills Program
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Managment
1900 E Street NW
Washington, DC 20415

1 Dr. Frank Withrow
U. S. Office of Education
400 6th Street SW
Washington, DC 20202

1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Non Govt

1 Dr. Earl A. Alluisi
HQ, AFHRL (AFSC)
Brooks AFB, TX 78235

1 Dr. Erling B. Anderson
University of Copenhagen
Studiestraedt
Copenhagen
DENMARK

1 1 psychological research unit
Dept. of Defense (Army Office)
Campbell Park Offices
Canberra ACT 2600, Australia

1 Dr. Alan Baddeley
Medical Research Council
Applied Psychology Unit
15 Chaucer Road
Cambridge CB2 2EF
ENGLAND

1 Dr. Isaac Bejar
Educational Testing Service
Princeton, NJ 08450

1 Dr. Warner Birice
Streitkraefteamt
Rosenberg 5300
Bonn, West Germany D-5300

1 Dr. R. Darrel Bock
Department of Education
University of Chicago
Chicago, IL 60637

1 Dr. Nicholas A. Bond
Dept. of Psychology
Sacramento State College
600 Jay Street
Sacramento, CA 95819

1 Dr. David G. Bowers
Institute for Social Research
University of Michigan
Ann Arbor, MI 48106

Non Govt

1 Dr. Robert Brennan
American College Testing Programs
P. O. Box 168
Iowa City, IA 52240

1 DR. C. VICTOR BUNDERSON
WICAT INC.
UNIVERSITY PLAZA, SUITE 10
1160 SO. STATE ST.
OREM, UT 84057

1 Dr. John B. Carroll
Psychometric Lab
Univ. of No. Carolina
Davie Hall 013A
Chapel Hill, NC 27514

1 Charles Myers Library
Livingstone House
Livingstone Road
Stratford
London E15 2LJ
ENGLAND

1 Dr. John Chiorini
Litton-Mellonics
Box 1286
Springfield, VA 22151

1 Dr. Kenneth E. Clark
College of Arts & Sciences
University of Rochester
River Campus Station
Rochester, NY 14627

1 Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

1 Dr. William Coffman
Iowa Testing Programs
University of Iowa
Iowa City, IA 52242

Non Govt

1 Dr. Meredith Crawford
Department of Engineering Administration
George Washington University
Suite 805
2101 L Street N. W.
Washington, DC 20037

1 Dr. Hans Cronbag
Education Research Center
University of Leyden
Boerhaavelaan 2
Leyden
The NETHERLANDS

1 Dr. Emmanuel Donchin
Department of Psychology
University of Illinois
Champaign, IL 61820

1 MAJOR I. N. EVONIC
CANADIAN FORCES PERS. APPLIED RESEARCH
1107 AVENUE ROAD
TORONTO, ONTARIO, CANADA

1 Dr. Leonard Feldt
Lindquist Center for Measurment
University of Iowa
Iowa City, IA 52242

1 Dr. Richard L. Ferguson
The American College Testing Program
P.O. Box 168
Iowa City, IA 52240

1 Dr. Victor Fields
Dept. of Psychology
Montgomery College
Rockville, MD 20850

1 Dr. Gerhardt Fischer
Liebigasse 5
Vienna 1010
Austria

1 Dr. Donald Fitzgerald
University of New England
Armidale, New South Wales 2351
AUSTRALIA

Non Govt

1 Dr. Edwin A. Fleishman
Advanced Research Resources Organ.
Suite 900
4330 East West Highway
Washington, DC 20014

1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

1 DR. ROBERT GLASER
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 Dr. Ross Greene
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

1 Dr. Alan Gross
Center for Advanced Study in Education
City University of New York
New York, NY 10036

1 Dr. Ron Hambleton
School of Education
University of Massechusetts
Amherst, MA 01002

1 Dr. Chester Harris
School of Education
University of California
Santa Barbara, CA 93106

1 Dr. Lloyd Humphreys
Department of Psychology
University of Illinois
Champaign, IL 61820

1 Library
HumRRO/Western Division
27857 Berwick Drive
Carmel, CA 93921

Non Govt

1 Dr. Steven Hunka
Department of Education
University of Alberta
Edmonton, Alberta
CANADA

1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105

1 Dr. Huynh Huynh
Department of Education
University of South Carolina
Columbia, SC 29208

1 Dr. Carl J. Jensema
Gallaudet College
Kendall Green
Washington, DC 20002

1 Dr. Arnold F. Kanarick
Honeywell, Inc.
2600 Ridgeway Pkwy
Minneapolis, MN 55413

1 Dr. John A. Keats
University of Newcastle
Newcastle, New South Wales
AUSTRALIA

1 Mr. Marlin Kroger
1117 Via Goleta
Palos Verdes Estates, CA 90274

1 LCOL. C.R.J. LAFLEUR
PERSONNEL APPLIED RESEARCH
NATIONAL DEFENSE HQS
101 COLONEL BY DRIVE
OTTAWA, CANADA K1A 0K2

1 Dr. Michael Levine
Department of Educational Psychology
University of Illinois
Champaign, IL 61820

Non Govt

1 Faculteit Sociale Wetenschappen
Rijksuniversiteit Groningen
Oude Boteringestraat
Groningen
NETHERLANDS

1 Dr. Robert Linn
College of Education
University of Illinois
Urbana, IL 61801

1 Dr. Frederick M. Lord
Educational Testing Service
Princeton, NJ 08540

1 Dr. Gary Marco
Educational Testing Service
Princeton, NJ 08450

1 Dr. Scott Maxwell
Department of Psychology
University of Houston
Houston, TX 77025

1 Dr. Sam Mayo
Loyola University of Chicago
Chicago, IL 60601

1 Dr. James A. Paulson
Portland State University
P.O. Box 751
Portland, OR 97207

1 MR. LUIGI PETRULLO
2431 N. EDGEWOOD STREET
ARLINGTON, VA 22207

1 DR. STEVEN M. PINE
4950 Douglas Avenue
Golden Valley, MN 55416

1 DR. DIANE M. RAMSEY-KLEE
R-K RESEARCH & SYSTEM DESIGN
3947 RIDGEMONT DRIVE
MALIBU, CA 90265

Non Govt

1 MIN. RET. M. RAUCH
P II 4
BUNDESMINISTERIUM DER VERTEIDIGUNG
POSTFACH 161
53 BONN 1, GERMANY

1 Dr. Peter B. Read
Social Science Research Council
605 Third Avenue
New York, NY 10016

1 Dr. Mark D. Reckase
Educational Psychology Dept.
University of Missouri-Columbia
12 Hill Hall
Columbia, MO 65201

1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007

1 Dr. Leonard L. Rosenbaum, Chairman
Department of Psychology
Montgomery College
Rockville, MD 20850

1 Dr. Ernst Z. Rothkopf
Bell Laboratories
600 Mountain Avenue
Murray Hill, NJ 07974

1 Dr. Donald Rubin
Educational Testing Service
Princeton, NJ 08450

1 Dr. Larry Rudner
Gallaudet College
Kendall Green
Washington, DC 20002

1 Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Non Govt

1 PROF. FUMIKO SAMEJIMA
DEPT. OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37916

1 Dr. Kazao Shigemasu
University of Tohoku
Department of Educational Psychology
Kawauchi, Sendai 982
JAPAN

1 Dr. Richard Snow
School of Education
Stanford University
Stanford, CA 94305

1 Dr. Robert Sternberg
Dept. of Psychology
Yale University
Box 11A, Yale Station
New Haven, CT 06520

1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN
THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305

1 Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

1 Dr. Brad Sympson
Office of Data Analysis Research
Educational Testing Service
Princeton, NJ 08541

1 Dr. Kikumi Tatsuoka
Computer Based Education Research
Laboratory
252 Engineering Research Laboratory
University of Illinois
Urbana, IL 61801

Non Govt

1 Dr. Maurice Tatsuoka
Department of Educational Psychology
University of Illinois
Champaign, IL 61801

1 Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

1 Dr. Robert Tsutakawa
Dept. of Statistics
University of MIssouri
Columbia, MO 65201

1 Dr. J. Uhlaner
Perceptronics, Inc.
6271 Variel Avenue
Woodland Hills, CA 91364

1 Dr. Howard Wainer
Bureau of Social SCience Research
1990 M Street, N. W.
Washington, DC 20036

1 DR. THOMAS WALLSTEN
PSYCHOMETRIC LABORATORY
DAVIE HALL 013A
UNIVERSITY OF NORTH CAROL
CHAPEL HILL, NC 27514

1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

1 DR. SUSAN E. WHITELY
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
LAWRENCE, KANSAS 66044

1 Dr. Wolfgang Wildgrube
Streitkraefteamt
Rosenberg 5300
Bonn, West Germany D-5300

Non Govt

1 Dr. J. Arthur Woodward
Department of Psychology
University of California
Los Angeles, CA 90024

1 Dr. Robert Woud
School Examination Department
University of London
66-72 Gower Street
London WCIE 6EE
ENGLAND

1 Dr. Karl Zinn
Center for research on Learning
and Teaching
University of Michigan
Ann Arbor, MI 48104